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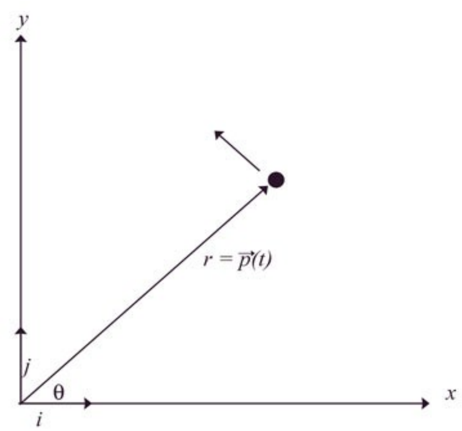
A Fudged Proof of $a=v^2/r$ on Youtube

by Miles Mathis

A reader who could not follow (or did not appreciate, at least) any of my previous critiques of this equation—where I analyze Newton, Maxwell, Feynman, and a current physics textbook—requested that I look at a video on youtube proving the equation. He said he was an engineer and that this is the way it is taught in engineering classes. He said that I would find analyzing this proof more "daunting." At first I answered, "Yes, I am sure it will be more daunting to analyze some guy on youtube than to analyze Newton, Maxwell, and Feynman." But eventually I saw a point in it. If some of my readers can't follow my other critiques, maybe they can follow this one.

As you might expect from something posted on youtube, this proof is fudged in even more ways the historical proofs. I agreed to write this paper not because the proof is really worth critiquing, but only because it is such a stellar example of what I have called fake calculus. This guy uses something that looks sort of like calculus to push his equations where he wants them to go, with no regard for rigor or logic, much less the basic rules of calculus and math in general. Of course, he didn't invent this sort of thing, he is just continuing a centuries-long cheat, one started by Lagrange when he differentiated the radius to create the Virial.

Here is the youtube proof, using his own diagram (which I have re-created).



p(t) = rcosθ(t)*i* + rsinθ(t)*j*
v(t) = dp/dt = r[-sinθ(t)](dθ/dt)*i* + r[cosθ(t)](dθ/dt)*j*

$\omega = d\theta/dt = \text{constant}$
$v(t) = dp/dt = -\omega r[\sin\theta(t)i - \cos\theta(t)j]$
$a(t) = dv/dt = -\omega^2 r[\cos\theta(t)i + \sin\theta(t)j]$
$a(t) = -\omega^2 p(t)$
$a = \omega^2 r$
$\omega = v/r$
$a = v^2/r$

That is basically it, minus the narration. Our narrator begins by calling the radius a "position vector," p(t), and he draws it as a vector and labels it as a vector, as you see. *There is no such thing as a position vector.* A position is scalar by definition, and so is a length. A vector implies motion, and we have nothing moving along this radius. Our orbiting particle out there is moving in an orbit, not outward along this radius. Therefore, the radius cannot be taken as any sort of vector in this sort of solution. He does the same thing with the velocity of the particle in orbit. He says this velocity is actually a scalar. He calls it a speed instead of a velocity. But neither speed nor velocity can be scalar, so I don't know what he is talking about. Both imply motion, motion implies direction, and direction requires a vector. *There is no such thing as a scalar motion*.

I will be told that speed is currently considered scalar, but that is only to allow current physicists to fudge proofs like this. All motion has a direction, whether that direction is explicitly stated or not, so motion is always a vector. *There is no such thing as motion without a direction*. So there is no physical difference between speed and velocity. They are straight synonyms. Just ask someone to give you a motion without direction. They tell you 10km/s, say, specifying no direction. But physics concerns real physical objects, by definition, so that speed must apply to some object. If that object is moving, it is moving in some real direction, whether you feel like telling me what that direction is or not. All motion is in some definite direction, by definition of motion, so that all motion is a vector, by definition. Motion is always a vector and position never is. If physicists don't currently understand that, it is only one more sign of dissolution.

If that is not clear in general, it should be clear in this problem at least. The orbiter must have a real direction at all times. It never has a nebulous "speed," in an indefinite direction. It always has a definite and *explicit* direction, and that direction is stated in the equations. Where? you ask. In the sines, cosines, and angles. At any given time, those angles will have definite values, which will give the orbiter a definite direction.

Also notice that his fake radius vector is drawn out from center, rather than in. The centripetal acceleration vector he is deriving must point in. How can you differentiate a position vector twice, thereby turning it 180 degrees? By what possible mechanism does our vector turn 90 degrees with each derivative? The position vector is out, the velocity vector is sideways, and the acceleration vector is in? How and why? Are we being told that gravity is actually an electrical vector, with some kind of righthand rule?

This is not quibbling, because it is this confusion that allows him to perform the magnificent mathematical cheat. I honestly believe this confusion is purposeful, and that these people tell you glaring falsehoods at the beginning of these fake proofs in order to stir your brain. If they confuse you with these upside-down definitions and backward vectors at the beginning, you will be too confused to see their upcoming cheats.

Now for the cheat. Notice that he labels the radius as both r and p(t). But is r = p(t)? I wrote the equality on the graph, but he doesn't. We aren't sure at first. Then we look at his first equation, and it tells us no. The position vector isn't simply r, it is a vector addition.

$p(t) = r\cos\theta(t)i + r\sin\theta(t)j$

To make us think that first equation can possibly solve the problem, he adds the function of time (t) and the little engineer's notation with *i*. But it clearly can't even begin to solve the problem, because the numbers going in and coming out are all constant. The length r is a constant, since we are looking a circular motion here. And so the vector additions of sin and cos are also constant. Nothing is changing but the angle, but our equation doesn't have any way of making the change in angle our velocity. In other words, you can't solve for a velocity, much less an acceleration, when your numbers aren't changing. Even before he begins his calculus manipulations, the equation has no chance of describing the motion of the particle. You can't solve an orbital velocity from just a radius, and that is all he has here. He has the radius and its x and y components, as you see. There is no way to solve for a velocity from that. It doesn't matter how many times you differentiate that first equation, it can't tell you anything about the problem at hand.

Current physics textbooks—like the one I critiqued in my other paper—seem to understand that, since they at least start with a tangential velocity vector. This is the way the proof has existed for centuries. That proof is also finessed, but nothing like this one. This guy tries to get an orbital velocity by differentiating components of the radius! Just study the form of his proof. He basically differentiates the radius to get the velocity, and then differentiates again to get the acceleration.

Just think about it. If you put different values for θ into his equations, could you find a speed? No, because whatever values you put in, the components will always add up to a length of r. And if you could solve that way, you wouldn't need both the sin and cos. You could solve with one or the other, since all you need is the angle in one place in the equations. It would be the angle giving us the velocity, not the sin or cos. Since r is a constant, the angle with either x or y, would give us a position at any moment. But we can't solve that way, because the angle and either x or y would give us a position, not a velocity. It would tell us the position at some moment, but what moment? Plus, this method of differentiating implies there is only one possible velocity for each radius, since the velocity is found by differentiating the radius. But we know that isn't true. In the orbital equations, you can have any given velocity at any given radius, depending on your field (central body). So the radius doesn't determine the velocity. The velocity is dependent upon both r and a, so you can't derive both a and v from r.

A reader will say, "Gosh, it seems like you should be able to derive circular motion, given that angle and a time." Well, yes, you can, though it isn't a derivation. If you are given how much angle in how much time, you are *given* the angular velocity, which by itself describes circular motion. And if you are given the radius, then you have that angular velocity at that radius. But none of that is a derivation. It is all just given data. And none of that tells you a necessary relationship between a and v and r, which is what we are seeking here. That has to be proven or derived, and it it can't be derived in this way.

Same thing for the acceleration. Let us say that you could find a velocity just by differentiating the radius. Can you then find an acceleration by differentiating that velocity? Isn't the velocity constant in circular motion? How can you differentiate a constant velocity and find an acceleration? You will say the angle is changing. Yes, but that change in the angle is what gave us the velocity. It can't give us the acceleration, too. Are you saying the change in angle is accelerating? It can't be, or the orbital velocity wouldn't be constant. The only thing that is changing here with time is the angle, and you can't get both a constant velocity and an acceleration from that one angle.

Obviously, if our narrator could derive a velocity by just differentiating the radius, he wouldn't need to bring the velocity into the proof by dishonest means. Since he *can't* differentiate the radius into a velocity, he gets it into his equations by just putting it in there by hand. What I mean is, look at how ω first enters the equations. It comes in on the wings of $d\theta/dt$, as you see.

$\omega = d\theta/dt$

But how did $d\theta/dt$ get in there? In got in by differentiating the position vector p. Unfortunately, $d\theta/dt$ is just notation, and it can't be taken as a velocity. It just means, "I have differentiated θ with respect to t here, so if you want to add dimensions later, remember you needs radians/second or something like that." In other words, it is just dimensional analysis, not a continuation of the derivative. It means "θ with respect to t" not "change in θ over change in time." The derivative of cosθ is -sinθ, not -sinθ times some velocity. So substituting ω for $d\theta/dt$ is a big bald cheat here. I have written a whole paper exposing this cheat, if you want to read more about it. Yes, physics and mathematics are now littered with this cheat, and you will see it everywhere, but that doesn't make it right.

Another problem is that this proof gets both the acceleration and the velocity by differentiating the radius. But historically, the two motions were separate. According to Newton (who was not overthrown by Einstein on this question) the sideways motion of the particle and the centripetal acceleration are separate. Not only are they not derivatives of one another, they aren't even functions of one another. Newton calls the sideways motion the body's "innate motion." It is innate because it isn't caused by the gravity field. There is no mechanism in a gravity field to cause tangential motion, and that is why Newton was required to keep it separate. Einstein doesn't have much to say about this*, but he certainly never overturned Newtonian mechanics on this question. It has simply been buried.

But regardless, you cannot solve by differentiating components of the radius, and neither Newton, Huygens, Maxwell, nor Feynman tried to do that. All of them recognized that you need at least two separate motions to solve, and that is why we are still taught the derivation that includes the tangential velocity. This proof here has no tangential velocity. The velocity here is found by differentiating the radius.

And that is why our narrator had to call the radius a vector in the beginning. That was strange, but he had to do it in order to differentiate it into a velocity. You can't differentiate a scalar or a length into a velocity, you can only differentiate a distance traveled into a velocity. So he has to pretend that the particle has somehow traveled out from the origin to where he has diagramed it.

Which brings us to more sleight of hand. After telling us that p(t) is *not* simply the radius, he comes back in line 7 and substitutes r for p(t). To make that substitution, he must be implying that $r = p(t)$. But if $r = p(t)$, then all that previous stuff was just fudge. If p(t) was always just a constant length, then all his previous manipulations are just magic. We now see that differentiating twice was just a trick to get his ω^2 that he needed.

Don't you find it strange that
$p(t) = r$
$v = r\omega$
$a = r\omega^2$

The position "vector" is equal to r, the velocity is just that times the angular velocity, and the acceleration is that times the angular velocity again. Those three equations, taken together, look fudged without any other analysis. We can write the last equation as a = vω. An angular velocity times a tangential velocity equals a centripetal acceleration? The tangential velocity is moving straight sideways; the angular velocity is moving mostly sideways, with only a component that is centripetal. How do you multiply those vectors to lose all sideways motion? Of course that is why we have to be told this equation is "scalar." It isn't scalar, but since it doesn't make any vector sense, we have to be warned off doing a vector analysis.

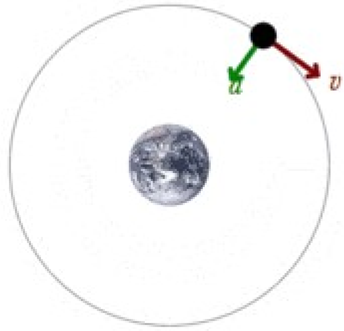
Finally, the clincher. To solve, this guy needed to *import* the equation v = rω. He couldn't have solved without that, could he? So that means he needs to know an angular velocity in order to calculate an orbital velocity, in order to calculate an acceleration. But without the proof of $a = v^2/r$—and its surrounding proofs—we don't know either the orbital velocity or the angular velocity. Remember, this proof is supposed to be showing us a way to find the orbital velocity. He is differentiating the radius in order to find the orbital velocity. The historical proof of this equation not only gave us the relationship of v to a, it gave us a way to find v. But by pulling in the equation v = rω from outside, he has gone circular: begging the question. He is assuming what he is meant to prove. The equation v = rω came from this proof, so you can't assume it in your derivation.

What I mean is, Newton's proof of this equation includes the proof that v = rω. Newton invents the calculus and goes to the limit to show how to get an orbital velocity from a given tangential velocity, which gives him the equation $a = v^2/r$. But this current fudge ignores all that and just takes the angular velocity as given, which means the orbital velocity is also given. That is circular, because now I can just ask for a proof of v = rω. I will be told that v = 2πr/t, but that isn't a proof either, it is just an assumption. It is assuming that curved motion is equivalent to a drawn curve; or that kinematics is equivalent to geometry. At least Newton tried to prove it. He was trying to derive the curved "velocity" from the straight line velocity at a point. So his proof isn't circular. This proof is circular, because it assumes it is given the curved velocity to start with.

If you still don't see my point, try this. I just showed that the current equation can be written like this: a = vω. Well, the variable ω already contains the centripetal acceleration, doesn't it? **Angular velocity is already circular motion!** So ω already has a centripetal component. This youtube proof is trying to derive that centripetal acceleration, so bringing ω in from outside is a cheat. The acceleration he ends up with at the end of the proof is not the acceleration he got from differentiating the radius twice, it is the acceleration that is already implied by ω. He is bringing in information from outside, so he is not proving anything. He is supposed to be showing where the centripetal acceleration comes from. In his proof, it is coming from the angular velocity v = 2π/t, which already contains it.

If ω already includes a, then a cannot equal vω.

I will show you what I mean. The "rigorous" way of solving circular motion was to give the orbiter two simultaneous motions. Newton gave it a tangential velocity and a centripetal acceleration.



That is still accepted (everywhere except youtube and engineering classes, I guess). Which means that the curved orbit is a vector addition of those two motions. Above, our narrator wrote r as a vector addition of its x and y components. Well, Newton did the same thing with the orbital motion. He gave it x and y components, sort of. The tangential velocity was the x component, and the centripetal acceleration was the y component. Since they are orthogonal, we can write a vector addition over any dt. Well, the angular velocity ω is already an orbital velocity: it is just written in radians per time rather than meters per time. That being true, the angular velocity is already a vector addition of the two components, including the centripetal acceleration. Like the orbital velocity, the angular velocity curves. An angular velocity already represents circular motion. Since all circular motion implies a centripetal force, ω already has a centripetal component. Since ω already contains a, you cannot come in later and multiply ω by something and claim to find a again (a cannot equal ba, unless b=1; a cannot equal b+a, unless b=0).

Think of it this way: If I say that my family is made up of 3 boys and 2 girls, I cannot come in later and multiply my family by b, finding the answer 2 girls. That is one of the basic rules of math. There is no possible integer value for b. You will do the math and say, "Sure, the answer is 2/5." But since that answer is less than one, it means you should have divided by b instead of multiplied. If you are given the sum 2 girls + 3 boys, and you want the answer 2 girls, you divide the boys from the girls. You don't multiply. If ω contains a, and you want a as your answer, you divide ω by something. You don't multiply. In fact, that is (part of) the answer here. I have corrected the equation v = rω. I have shown that it should be $v = \sqrt{[(\omega^4/4r^2) + \omega^2]}$. For very small particles like quanta, we can write the equation as v = ω/r. It has been wrong for centuries, which is what has required all these fudges (including the fudged moment of inertia).

In fact, we have conspicuous and longstanding data proving I am right. According to the current equation v = rω, a greater radius should give us a greater velocity. According to my new corrected equation, a greater radius should give us a lesser velocity. What do we find?

velocity of Mercury 48km/s
velocity of Venus 35km/s
velocity of Earth 30km/s
velocity of Mars 24km/s
velocity of Jupiter 13km/s

Need I go on?

Yes, to top it all off, I have shown in other papers that the equation $v = r\omega$ is false. That v is currently defined as the tangential velocity, but the tangential velocity cannot be written as $2\pi r/t$. That ratio is a curve, and the tangential velocity is not. Besides, our narrator here doesn't want a tangential velocity in his equation. He wants an orbital velocity, so that he can solve by just inserting a circumference of orbit over time of one revolution. And if there were any possible way to differentiate a radius into a velocity, it would have to be an orbital velocity, not a tangential velocity. He even seems to understand that, since he draws his particle with a curved motion. He never draws a tangential velocity. Current physics has no interest in a tangential velocity, and it looks like it is even trying to jettison it from the centuries' old proof of $a = v^2/r$. This youtube proof buries the tangential velocity completely, never even mentioning it. But, as I have shown, that is not an improvement on the old math. The old math was finessed, the new math is triple finessed. The new derivation is not clearer, it is muddier than mud and twice as dirty.

Of course I know this newest critique won't convince many people. If they couldn't follow my other analyses, they probably won't be able to follow this either. Others won't *want* to follow it, so they will pretend they can't follow it even if they can. It is so much easier just to keep what we have, I know. It is what we were taught, it has been gospel for centuries, and—if you don't study it too closely—it all seems to fit together. So why question it?

Some think I write these papers just to be contrary or to be novel. I don't. I write them because I want to solve these problems properly. I want the right answer, not just the given answer. If I find a problem, I drag it out into the light.

Others will say, "You *can't* be right. If these problems really existed, someone would have mentioned them before. This equation has passed muster with centuries of geniuses." As if that is a good answer. Galen's anatomy passed muster with centuries of geniuses, too, until Rudbeck and others came along. Aristotle's celestial mechanics passed muster with centuries of geniuses before Copernicus and others came along. Newton's mechanics was thought to be complete and infallible until Einstein came along. I could go on for weeks, just listing the top examples.

I am not claiming to be smarter than all the geniuses in history who missed this. I just happened to see it, I don't know why. Call it luck, if you wish. All I am claiming is that the problem exists.

I suppose I am claiming to be more honest than mathematicians and physicists and engineers who keep fudging these proofs. I can't believe they think they are doing real math, which is why I get testy sometimes. This late in history, "top" mathematicians and physicists should at least understand some of the basic rules of logic and consistency. A lot of them seem not to, and I chalk this up to dishonesty instead of ignorance. Ignorance is no sin, and I don't mind teaching those who are interested in the truth but haven't yet found it; but dishonesty really irks me.

I will answer one question in closing. One of my clever readers might say, "Wait, don't you let the radius be a velocity in one of your papers? The ones on pi? You say that if we bring time back into the analysis, then the radius becomes a velocity." Yes, I do, but that is completely different than this. I do that only to match the length of the tangent to the length of the radius. I never call the radius a position vector. I never imply that an orbiter moves along the radius or any component of the radius. I never differentiate the radius or any component of it. I don't imply—and my math doesn't imply—that the tangent is a derivative or a function of the radius. I simply say, "What if we let the radius equal the tangent in length?" After the length of the tangent is set, I never treat the radius as a vector after that. I do this to take us to $1/8^{th}$ of the circle, where I can show some simple math that solves this problem. I think that anyone who studies my math and analysis will see that it has nothing in common with the above proof.

*In fact, Einstein explicity confirmed in his tensor calculus proof of general relativity that there was no force at a tangent in the gravitational field. This means that any motion sideways must be independent of the field.