The Two-Envelopes Paradox

by Miles Mathis

A reader recently sent me a question about this problem, which—believe it or not—I hadn't seen before. She linked it to quantum paradoxes, whereby bad statistical analysis allows modern physicists to create problems and paradoxes where there are none. It is in these gaps that they play their Modern games. She is absolutely correct in this.

The two-envelope problem is very simple. You have two envelopes and are told only that one contains twice as much money as the other. Having no way to calculate which is the better choice, you choose one randomly (or are simply given one at random). After that choice, you are given the opportunity to switch. Some mathematicians have run the odds, showing it is to your benefit to switch. How can that be?

Well, to write the equation, they start with these assumptions. Assume the envelope you have chosen contains $20. There are two possibilities and only two. One, you have the larger sum, so the other envelope contains $10. Two, you have the smaller sum, so the other envelope contains $40. Using that assumption, it is indeed possible to write probability equations indicating it is better to switch. The reason is those equations contain three values, which are x, 2x, and x/2. However, since in real life there are never three possibilities, the math is wrong.

The math is wrong because the problem is stated incorrectly. The problem should be stated like this: let us say one of the envelopes contains $20. That envelope is *either* the larger or the smaller, *but not both*. Therefore, the amounts in the envelopes are either $20 and $10, *or* $20 and $40, *but not both*. There is only one reality here, not two. The actual state is a single state. If the actual state is $10 and $20, then $40 doesn't enter the equations. If the actual state is $20 and $40, then $10 doesn't enter the equations. So you have to write the equations in terms of x and 2x, *or* x and x/2. You cannot write the equations as a function of all three. If you do, you have made a terrible mathematical mistake, it is that simple.

The fake paradox is created by using three terms in a problem where there are only two.

This paradox is like Zeno's paradoxes, which I have shown are stated wrongly on purpose, to test your ability to spot flaws in the postulates.

Admittedly, the above solution is "the common one." I am not discovering anything new here (although I may be stating it a bit more clearly). However, this fake paradox has, for some reason, gotten a lot of attention from professional mathematicians recently. Although the answer is simple, a lot of people are doing their best to muck it up. Why? At Wikipedia, we are told this:

No proposed solution is widely accepted as correct. Despite this it is common for authors to claim that the solution to the problem is easy, even elementary. However, when investigating these elementary solutions they often differ from one author to the next. Since 1987 new papers have been published every year.

So a lot of "important" people, including the academic writers of Wikipedia, are telling you that the common solution is oversimplified, and that the problem is deeper than it appears. Why would they do that? Are they stupid? No, they are spreading confusion on purpose, because it benefits many subfields of mathematics and physics to spread that confusion. Thousands of big names have set up shop in the gap created by this fake paradox and other similar paradoxes, and if those paradoxes evaporated, they would be out of a job. Just one such paradox is entanglement, another manufactured problem that has buttered a lot of people's bread over the past several decades. I beg you to notice the similarity between the envelope problem and Schrodinger's cat. You just saw how confusion is inserted into the envelope problem by trying to divert the audience away from the obvious fact that there is only one actuality here. Those who muck up the problem either misdirect you away from that question, or they actually put the question in the open and then lie about it. They deny that there is only one reality. They put on the pettifogger's hat and try to convince you it is more interesting and creative to assume that there are an infinity of realities. They say that before we open the envelopes, the numbers could be $20 and $10, $20 and $40, *and* any other multiples of 2. Since we don't know, the possibilities are endless. This is where we get the many-worlds hypothesis of Hugh Everett.

Notice how I italicized the word *and* just above. That is where the switch is made. Notice it should be, "before we open the envelopes, the numbers could be $20 and $10, $20 and $40, *or* any other multiples of 2." To get the many-worlds hypothesis, you need to write it with the *and*. But since the correct way of stating the problem is with the *or*, **only one** of the states exists in any one problem. Not all of them at once, but only one of them.

Again, these people are just spreading confusion on purpose. They are pretending they don't know the difference between the set-up and the statistics, although of course they do. No one is that stupid, therefore they must be lying. Remember, someone had to set up the problem. Someone had to put an amount of money in each envelope. Money doesn't just insert itself into envelopes. So, although our choosers might not know the real amounts, someone does. That person knows there are two amounts in the two envelopes, not three or infinity. But even that is not stating this simply enough. It has nothing to do with knowledge, or who knows what. It has only to do with logic. As I just said, the two envelopes contain two and only two amounts, therefore the probability math can never contain three variables.

To say it another way, the two envelopes *can* contain cash amounts of any multiple of 2, but they *do* contain only one multiple of 2. No many-worlds exist here. Only one world exists. Those who are spreading confusion here are doing it by conflating *can* with *do*. They are trying to make you think that because there are many possibilities, those possibilities *exist*. But they don't exist. Or, they exist only as potentials. These people are playing you on the word *exist*. Because you can say, "the possibility exists," they are telling you all those possibilities *exist at the same time* in some real world. But they don't. Possibilities don't exist like that. Possibilities exist as ideas, in your head or on some sheet of paper. But the money in the envelopes exists in a different way. It exists as real objects in the real world. And the money exists in only one state throughout the problem. Once the cash is hidden within the envelope, it does not suddenly enter an indefinite or infinite state. Everybody knows that. Your mother knows it, your children know it, your babysitter knows it, and these fake mathematicians know it.

Despite that, Bayesian analysis is brought in to confuse the issue. At the Royal Society, they say this:

While the Monty Hall problem is tractable to elementary analysis, the two-envelope problem requires advanced techniques akin to those found in McDonnell et al. (2008).

Of course they would say that, since that quote is from a paper by, yes, Mark McDonnell. But since the Monty Hall problem concerns three doors, with added complexity, how can it be "tractable to elementary analysis," while the Envelopes problem, which concerns only two envelopes, must require advanced techniques? Since Mark McDonnell mentions the Monty Hall problem, we will see below what Marilyn vos Savant has to say about it. Marilyn embarrassed thousands of math PhD's on the Monty Hall problem, so we will see if she agrees that this requires advanced techniques.

But first, let us see why Bayesian analysis might be required. We are told at Wikipedia:

Here the ways the paradox can be resolved depend to a large degree on the assumptions that are made about the things that are not made clear in the setup and the proposed argument for switching. The most usual assumption about the way the envelopes are set up is that a sum of money is in one envelope, and twice that sum is in another envelope. One of the two envelopes is randomly given to the player (envelope A). It is not made clear exactly how the smaller of the two sums is determined, what values it could possibly take and, in particular, whether there is a maximum sum it might contain. It is also not specified whether the player can look in Envelope A before deciding whether or not to switch. A further ambiguity in the paradox is that it is not made clear in the proposed argument whether the amount A in Envelope A is intended to be a constant, a random variable, or some other quantity.

All these extra issues don't make any difference at all, as you see if you think about it. Say the chooser is allowed to open his envelope and see the amount in it. Would that make any difference? No. That knowledge can't help you make a decision, so it can't enter the odds. Say you open your envelope and find $20. All you know is that the other envelope either has $10 or $40. But since there are no logical odds you can put on $10 or $40, that doesn't help you at all. Your knowledge of whether you have the higher or lower amount is still zero, which *prevents* you from assigning any new odds. You are no better off than you were before.

The question of whether the values are constants or random variables is also not pertinent. It is just more misdirection. Obviously, the values are *predetermined values*, since there is no physical way to insert variables into envelopes. Since predetermined values are even more solid than mathematical constants, this realization short-circuits this attempt at misdirection. A mathematical constant, even though not a variable, can still take different values, as we have seen. It can be defined as a constant *within* rolls, but can vary in the possibilities. But a predetermined value is more constant than any mathematical constant, since it cannot vary under *any* circumstance. Once the envelopes are filled, it is one value, constant across all possible mathematical manipulations.

So the fact that this problem has re-entered the playing field is actually a political phenomenon, not a mathematical one. No one with an IQ over 80 really believes this problem suddenly became more interesting or more difficult in 1987. So why did it re-enter the literature big-time in 1987-1989? It did so as back-up to Schrodinger's cat problem, superposition, entanglement, and the other manufactured paradoxes of physics, which were in some danger at that time. This was just after the arrival of String Theory, and the theorists there were re-using the old fake paradoxes in their theories to manufacture even more novel interactions. Problem is, they were getting some backlash from the few remaining people in math and physics who retained some scruples. We may assume that some of this backlash was so pointed it threatened to bring down superposition and entanglement. Well, since superposition and entanglement had been found to be such moneymakers—not only for quantum physicists but for all the mainstream physics and science journals—this backlash had to be countered. Therefore, the math departments were called in to increase the levels of confusion and misdirection, in any way they could. They were instructed to publish a barrage of papers, to use as complex terminology and symbolism as they could, to import as many forms of Modern analysis (like Bayesian analysis) as possible, and to avoid making sense. This they have done.

When I first got the email that led to this paper, I told the reader I was too deep in other problems to look at it. I recommended she go to Marilyn vos Savant, who I suspected had already answered this in her *Parade* column. I trust Marilyn's judgment on questions like this and have defended her in the Fermat brouhaha. However, I soon surfaced long enough to take a peek at the problem as given at Wikipedia, and I saw the answer immediately. Since I also saw that my quick solution was the "common" one, I thought I would leave it at that. I didn't think my two cents needed to be thrown in here. However, reading a bit further on Wikipedia, I soon saw how this tied into Schrodinger's cat and entanglement. I also began to understand the timing of the re-entry of this problem into the literature, giving me something to say on the problem that I don't think anyone else is saying.

I will conclude by re-printing what Marilyn said in *Parade* in 1992 [Sept. 20]:

While it appears as though you should switch, because you have an even chance at $200 vs. $50, which any gambler would grab, it actually makes no difference at all. Those even chances would apply only if you could choose one of two *more* envelopes, one with $200 and one with $50. As it is, there is just one more envelope sitting there, with either twice the amount you have just seen or half of it. And you knew that would be the case before you even started. So when you opened the first envelope, you didn't gain any information to improve your chances. This can be illustrated by noting that the logic that causes you to switch (because you appear to have an even chance for $200 vs. $50) will lead you to switch every time (no matter what you find in the first envelope), making the second envelope just as randomly chosen as the first one!

In other words, to switch, you would need *three* envelopes, which would give you three variables, which would give you the odds you need to justify the switch. But without three envelopes, you have no reason to switch. Notice that Marilyn doesn't need any advanced techniques to come to this conclusion. She states the case in six sentences, with no variables. Although I found her answer after I had come to my own, I was gratified to find we reached the same conclusion. In the simplest problems, you often worry the most that your first reaction is wrong, because it seems too easy. Although I rarely need or look for confirmation, I was glad to find it here.

Also notice the difference between Marilyn's logic and the mainstream lack of it. She uses the created paradox as a signal that you can't go there. She doesn't see the paradox of switching everytime as something to embrace and exploit. She doesn't see it as a welcoming data hole where she can set up shop and start blowing smoke. She sees it as sign to stop, not as a sign to go on.

Some will say, "Well, she appears to beat you for brevity, as well as politeness. She doesn't call anyone stupid or a liar. Why not follow her example?" I can answer that as well. Although her answer is right enough for *Parade* magazine, it leaves something to be desired. For starters, notice she is answering the problem with the additional complication of peeking. She is absolutely correct in what she says, but her analysis doesn't apply if you don't peek. The odds remain the same, but the reasons are slightly different. The even chances don't come up at all, as you see. Without the peeking, you don't switch because there is no way to write odds that would indicate you should. I think it helps to show why you can't write three-way odds for two envelopes. It also helps my analysis to show how the other guys push this problem. Showing you the several ways they cheat, what with the 3-for-2 cheat, the *and/or* cheat, and the "existence" cheat, ties this more clearly to Schrodinger's cat and entanglement. Tying it to physics and String Theory helps you to understand the why and when of the cheat. If you can understand *why* mathematicians are pushing equations, it often helps to see *how* they are pushing.

As for my lack of politeness, I feel the time for that is over. Marilyn tried politeness and it didn't get her anywhere. These are the people (or the same sort of people) that jumped all over her in the 1990's. Again, it is helpful to know the politics. Marilyn is not considered to be a professional mathematician, simply because she doesn't hold down some chair somewhere. The fact that she solves problems better than almost anyone doesn't mean anything to professional mathematicians, it only annoys them. In a word, they are envious of her intelligence. The fact that she made hoards of them look like idiots with the Monty Hall problem and other problems only salted the wound. In 1990, her correct solution was contested by 1000's of PhDs, including top academics. As Wikipedia admits, "Paul Erdős, one of the most prolific mathematicians in history, remained unconvinced until he was shown a computer simulation confirming the predicted result." He wasn't the only one. Some of these academics are *still* attacking Marilyn in forums and at Amazon.com and so on. They continue to attack her on her Fermat statements as well, although she was right the first time. She eventually backed down. I didn't.

She probably thinks those who attack her just honestly disagree. Or perhaps she would admit they are envious and are nasty people. But I don't think she has considered the possibility that they are being *paid* to pretend to disagree with her and with all straightforward analysis. Last time I emailed her (quite a while ago), she didn't seem open to that possibility. It may be that the events of the last decade have changed her mind.

It is now known that many people *are* paid to create destabilization in many fields. This is not a theory, much less a conspiracy theory. Documents have been declassified proving it. Ex-agents have admitted it. It has been proven in the arts and sciences as well as in politics. The world is full of liars, and many of them are paid liars. I didn't call anyone stupid in this paper, notice. I said they *couldn't be* stupid enough to mess this up as badly as they are messing it up. But I did call them liars. The evidence tells us unequivocally that lies are being told on purpose, and a lie requires a liar. Just as envelopes do not fill themselves, lies do not tell themselves.

Entanglement is not a mistake. It was not caused by an honest lack of understanding. The current interpretation of Schrodinger's cat is not a mistake. It was not caused by a lack of understanding. It is purposeful misdirection, and the misdirection has been embraced and promoted because it creates jobs. It was found that it made a more salable story than the truth. People will read about entanglement and spooky forces and many-worlds and the observer effect until the cows come home, but they only yawn at the truth. Straightforward sensible analysis amuses most people like dry white toast or GMO tomatoes. It just doesn't have enough flavor. They have been raised on a constant diet of jolt cola and twinkies, and logic simply isn't sugary enough for them. Consistency isn't spicy enough. Rigor is *boring*. They need a bit of magic and miracle in every paragraph, a rabbit pulled from the void in every equation.

But these problems and equations aren't being pushed just to appeal to the great unwashed and untutored. Theories like entanglement continue to be pushed because the greater part of quantum mechanics rests on these theories. If these paradoxes are killed, it is not only jobs that are killed and glossy covers of *Scientific American* that are killed, it is the entire foundation of quantum mechanics that is killed. If physics loses entanglement and superposition and tunneling and these other paradoxes, it crumbles into a heap. If quantum mechanics falls, then QED and QCD fall, the standard model falls, and the theory supporting the Large Hadron Collider and other major projects fails. So you see why the mainstream might be interested in protecting entanglement, even if it means lying their asses off.

Of course, there is an alternative. I have shown that quantum mechanics can easily be rebuilt on firmer foundations, keeping all the data and a good part of the theory, while losing entanglement, spooky forces, virtual particles, borrowing from the vacuum, and all the other schist that has accumulated over the years. In this rebuilding, new jobs will be created and new projects will be required. And since these projects will be based on firm theory and equations, they will be far more productive in every way. Physics is now stuck, but once these corrections are made, it will begin to move again. Progress always creates more jobs than stasis.